Random Fourier Features based Post-Distortion for Massive-MIMO Visible Light Communication

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***Abstract*—Massive multiple input multiple output (m-MIMO) visible light communication (VLC) has emerged as a viable technology to enhance the throughput of an existing VLC based systems in order to serve the high speed data requirement for beyond 5G and 6G communication systems. However, performance of m-MIMO VLC is severely impaired by (1) columns of m-MIMO VLC channel are highly correlated which makes the channel matrix ill-conditioned, and (2) nonlinear transfer-characteristics of light emitting diode (LED). The aforementioned channel impairments collectively degrade the overall bit error rate (BER) at the receiver. In this paper, finite memory budget adaptive precoder is proposed to decrease the channel correlation/condition number of MIMO channel matrix. Reproducing kernel Hilbert space (RKHS) based post-distorters have been proposed in the literature for VLC which rely on growing dictionary of kernel evaluations, and hence it is difficult to practically implement them under finite memory budget. In this paper, random Fourier features (RFF) based kernel least mean square (RFF-KLMS) algorithm is proposed for post-distortion over m-MIMO VLC channels which alleviates the requirement of dictionary and facilitates post-distortion under finite memory budget. Computer simulations performed over m-MIMO VLC channels show that the proposed RFF based algorithm exhibit similar BER performance with reduced computational complexity as compared to the dictionary based post-distortion algorithm.**

***Index Terms*—VLC, MIMO, kernel, Hilbert space, random Fourier features, post-distorter, RFF-KLMS.**

I. Introduction

Visible light communication (VLC) is evolved as a viable supplement to the current radio frequency (RF) technology [1], [2] due to the several desirable features such as: (1) large bandwidth (of the order of THz), (2) low cost as VLC uses existing lightning framework which makes it suitable to achieve twofold goal of illumination and communication, (3) green since it uses less energy, (4) secure and free from electromagnetic interference which makes VLC suitable to be used in hospitals, aircrafts and war-sites where RF cannot be used, and (5) high signal to noise ratio (SNR) due to the illumination order of several hundred lux [3]. To increase the throughput of single-input single-output VLC systems, the use of massive array of transmitting light emitting diodes (LEDs) and receiving photodiodes (PDs) have been suggested in the literature [4]–[6], which is termed as massive multiple input multiple output (m-MIMO) VLC system. [4].

Performance of a m-MIMO VLC system is severely degraded by the following two major factors: (1) high condition number of the m-MIMO VLC channel due to similar channel gains between the LED and two closely placed PDs in an indoor VLC scenario, which results in amplification of noise at the receiver upon using channel inversion based detection algorithms like zero forcing and minimum mean square error (MMSE), and (2) nonlinear transfer-characteristics of LED which results in nonlinear distortions, and deteriorate the bit error rate (BER) performance.

A. Related Works:

Various transceivers have been designed for multiple input single output (MISO) and MIMO VLC [7]–[10]. However, combined problem of MIMO channel correlation and LED's nonlinearity is not addressed in [7]–[10]. In addition to Tiknonov regularization and preconditioning techniques, singular value decomposition (SVD) based adaptive precoders had been proposed in [5], [6] to reduce the condition number of the overall m-MIMO VLC channel matrix. However, LED's nonlinearity is ignored in [5] for optimization of precoding index, and hence delivers suboptimal performance for nonlinear VLC systems. To circumvent this sub-optimality, authors in [6] had considered LED's nonlinearity for optimization of precoding index by mapping the received observations from original input space to high dimensional reproducing kernel Hilbert space (RKHS). However, RKHS methods rely on growing dictionary of observations which increases the overall computational complexity and storage requirements.

Further, various pre-distorters have been proposed in the literature for mitigating LED's non-linearity [11]–[13]. Authors in [11] have proposed a linear normalized least mean square (NLMS) based pre-distortion algorithm for VLC which delivers suboptimal performance for nonlinear systems. Hence, in [12], [13], *R. Mitra et al.* have proposed nonlinear Chebyshev-NLMS based pre-distortion algorithm for mitigating LED's nonlinearity. However, pre-distorter methods depend upon the presumption of perfect feedback to the transmitter which is not practical, thereby calling for post-distortion techniques. In the existing literature, post-distortion methods have been divided into the following two major categories [14]: (1) polynomial series (like Volterra series, Volterra-DFE, and Hammerstein) based post-distorters [15], [16], and (2) RKHS based post-distorters (like kernel least mean square (KLMS) [17], [18], and kernel minimum symbol error rate (KMSER) algorithms [19]–[21]). Volterra-DFE/Hammerstein based post-distorters are computationally complex, requires large number of iterations to converge, and impaired by approximation error due to abrupt truncation of Volterra series till lower (second/third) order terms [14].

RKHS based post-distorters have been preferred over the polynomial series based methods since they guarantee unique representation of a wide class of non-linearities, and facilitate computational simplicity achieved through online dictionary based sparsification techniques like novelty criterion (NC), modified NC, quantization criterion, coherence criterion, and spherical radius criterion [14], [17]. These sparsification techniques adopt a suitable criterion to decide whether the incoming observations is to be included or rejected in the dictionary. Although these sparsification techniques significantly reduce the dictionary size, however they require significant computational resources to perform sequential search over all the existing elements of the dictionary [22], which limits the practical applicability of dictionary based RKHS based post-distorters.

Contributions: The primary contributions of this paper are outlined as follows:

- In this paper, we propose an index based SVD based adaptive precoder to decrease the corelatedness among columns of m-MIMO VLC channel matrix. Further, precoding index is optimized by using minimum symbol error rate criterion that considers LED's nonlinearity by mapping the received observations to a finite dimensional Euclidean space, instead of RKHS (whose dimension is larger than the dimension of input space) through a randomized feature map, which facilitates implementation of proposed adaptive precoder under finite memory budget.
- Next, random Fourier features KLMS (RFF-KLMS) based post-distortion algorithm is proposed for m-MIMO VLC to mitigate LED's nonlinearity, where feature map of the Gaussian kernel is approximated by using random Fourier features of the kernel's Fourier transform [22]. Hence, the proposed RFF-KLMS based post-distorter does not require sparsification, and facilitates post-distortion under fixed and finite memory budget, which makes the proposed algorithm viable for practical implementation.

Remainder of this paper is structured as follows: the channel and system model for m-MIMO VLC is desribed in Section-II. The proposed algorithm is described in Section III. Simulations are given in Section IV, and lastly Section V concludes the paper.

Notations: Small/capital boldface symbols/characters are used to represent vectors/matrices. Real part of $(\cdot)^R$ denotes real part of a complex quantity $(\cdot)$ while $(\cdot)^I$ denotes imaginary part of any complex quantity. $\odot$ denotes Hadamard product. $(.)^T$, and $(.)^\dagger$ denote transpose, and pseudo-inverse, respectively. $||\cdot||_{\mathbb{C}^D}$ denotes norm in finite dimensional Euclidean space $\mathbb{C}^D$, and inner product in $\mathbb{C}^D$ is denoted by $\langle\cdot,\cdot\rangle_{\mathbb{C}^D}$.

II. Channel and System Model

The proposed system model is shown in Fig. 1. First, the data bits are modulated through carrier-less amplitude and phase quadrature amplitude modulation (CAP-QAM) scheme to generate modulated symbol vector $\mathbf{s}_k \in \mathbb{C}^{N_t\times 1}$, where N_t denotes the no of LEDs (subscript $(\cdot)_k$ denotes symbol at k^{th} time instant). Details of CAP-QAM modulation scheme is given in [23], [24]. Precoder matrix $\mathbf{P}$ is multiplied with the input signal before transmission. Further, DC bias $\mathbf{b}$ is added to the precoded signal so that the symbols are mapped to first quadrant in order to ensure LED to be biased in forward bias regime [19]. Next, DC-biased symbols are transmitted by LED through m-MIMO VLC channel. The overall transmitted signal by the LED can be written as: $\tilde{s}_k = \mathbf{P}\mathbf{s}_k + \mathbf{b}$. The LED's nonlinearity is characterized by Rapp's model as [25]

$$f(s_k) = \begin{cases} \frac{(s_k - V_{\text{th}})}{\left(1+\left(\frac{(s_k-V_{\text{th}})}{V_{\max}}\right)^{2q}\right)^{\frac{1}{2q}}} & s_k \geq V_{\text{th}} \\ 0 & s_k < V_{\text{th}} \end{cases} \tag{1}$$

where $V_{\max}$, V_{th}, and q is saturation voltage, threshold voltage of LED, knee factor, respectively.

The indoor m-MIMO VLC channel matrix is denoted by $\mathbf{H} \in \mathbb{R}^{N_r \times N_t}$ (where N_r denotes the no of PDs). The channel coefficients/elements h_{ij} between j^{th} transmitter LED and i^{th} receiver PD [4] is given as [4]

$$h_{ij} = \begin{cases} \frac{A_e(p+1)}{2\pi d_{ij}^2}\cos^p(\phi)\cos(\theta) & 0 \leq \theta \leq \theta_c \\ 0 & \theta > \theta_c \end{cases} \tag{2}$$

where θ_c represents the field-of-view (FOV) semi-angle of the PD, θ is the angle of incidence from the PD axis, ϕ indicates the angle from the LED axis, the parameter $p = -\frac{\ln 2}{\ln \phi_{1/2}}$ is known as the Lambert's mode order and $\phi_{1/2}$ is the LED's semi-angle at half power , A_e is the effective collection area of the PD, and d_{ij} is the distance between j^{th} transmitter and i^{th} receiver.

The received symbol vector at k^{th} time instant by the PD array can be written as

$$\mathbf{r}_k = \mathbf{H}f(\mathbf{P}\mathbf{s}_k + \mathbf{b}) + \mathbf{n}_k \tag{3}$$

where $\mathbf{n}_k$ (which consists of thermal noise and ambient light [26], [27]) is approximated as Gaussian distribution $\mathbf{n}_k \sim \mathcal{CN}(0, \sigma_n^2\mathbf{I}_{N_r})$. Next, received symbols are applied as an input to the proposed RFF-KLMS based post-distorter to recover the transmitted signal.

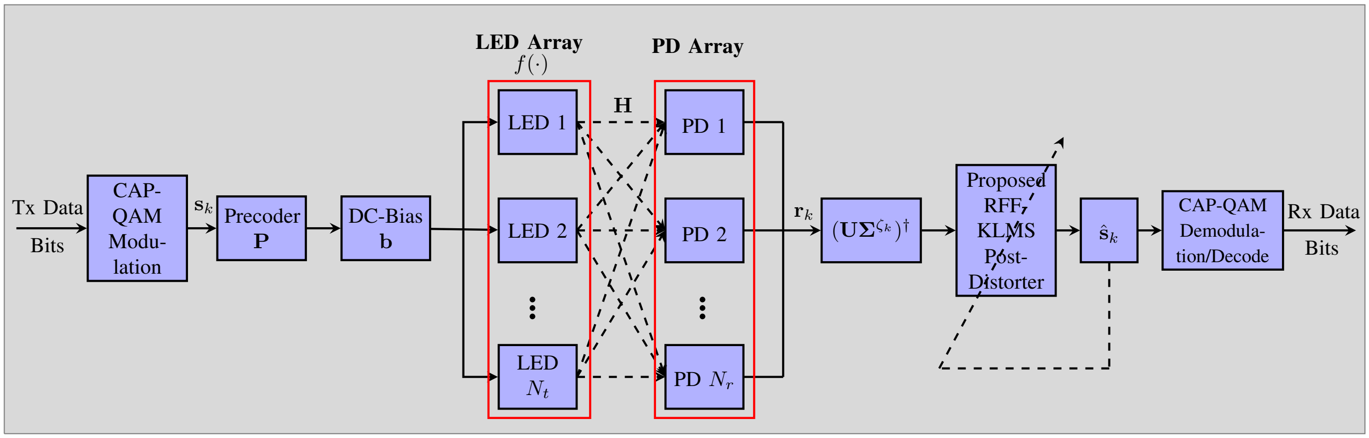


Fig. 1. Proposed model for m-MIMO VLC system

III. Proposed Algorithm

This section describes the proposed fixed size adaptive precoder and RFF-KLMS (AP-RFF-KLMS) based post distorter for ill-conditioned and nonlinear m-MIMO VLC channel.

A. Proposed Fixed Memory Budget Adaptive Precoder

In this subsection, the fixed memory budget adaptive precoder is proposed for reducing the m-MIMO VLC channel correlation/condition number. As in [6], input signal is first multiplied by the precoder matrix (denoted by $\mathbf{P}$) before transmission as

$$\mathbf{P} = \mathbf{V}\Sigma^{\zeta_k - 1} \tag{4}$$

where $\mathbf{U} = [\mathbf{u}_1,\ \mathbf{u}_2,\ \ldots,\ \mathbf{u}_r]^T$, $\mathbf{V} = [\mathbf{v}_1,\ \mathbf{v}_2,\ \ldots,\ \mathbf{u}_r]^T$ are obtained by performing truncated SVD on $\mathbf{H}$, where each $\{\mathbf{u}_i\}_{i=1}^r \in \mathbb{R}^{N_r}$ and $\{\mathbf{v}_i\}_{i=1}^r \in \mathbb{R}^{N_t}$ are left and right singular vectors of $\mathbf{H}$, $\mathbf{\Sigma} = \text{diag}(\rho_1,\ \rho_2,\ \ldots,\ \rho_r)$, $\{\rho_i\}_{i=1}^r$ are first r (where r is rank of the channel matrix) singular values of the channel matrix $\mathbf{H}$, and $\zeta_k \in [0, 1]$ is the precoding index at k^{th} time instant. As inferred from (3) and (4), and from [6], it can be observed that precoding index ζ_k affects the performance of the precoder. Hence, precoding index ζ_k is optimized to compromise between the condition number and peak to average power ratio (PAPR) [6].

Hence, based on received observations $\mathbf{r}_k$, optimization of ζ_k is performed via minimum symbol error rate (MSER criterion) in finite dimensional Euclidean space $\mathbb{C}^D$ (instead of RKHS), which can be written as

$$\begin{aligned}&\min_{\zeta_k}\ \ ||\mathbf{\Theta}_k - \mathbf{\Theta}_{k-1}||^2_{\mathbb{C}^D}\\ &\text{s.t.}:\ \boldsymbol{\mathcal{I}}_k(\zeta_k) = 0 \text{ and } 0 \leq \zeta_k \leq 1,\end{aligned} \tag{5}$$

where $\boldsymbol{\mathcal{I}}_k(\zeta_k) = (\tanh[\beta(\mathbf{\Delta}^R(\zeta_k) + 1)] + \tanh[\beta(\mathbf{\Delta}^R(\zeta_k) - 1]) + j(\tanh[\beta(\mathbf{\Delta}^I(\zeta_k) + 1)] + \tanh[\beta(\mathbf{\Delta}^I(\zeta_k) - 1])$ is the MSER constraint [21], [28], $\mathbf{\Delta}(\zeta_k) = (\langle\mathbf{\Theta}_k, \mathbf{r}_k^{'}\rangle_{\mathbb{C}^D} - \mathbf{s}_k)$, $\mathbf{r}_k' = (\mathbf{U}\Sigma^{\zeta_k})^{\dagger}\mathbf{r}_k$, and $\mathbf{\Theta}_k$ is the weight matrix in $\mathbb{C}^D$. The optimization problem in (5) can be solved by formulating the Lagrangian using quadratic penalty function as [21], [29]

$$\mathscr{L}(\zeta_k) = \|\mathbf{\Theta}_k - \mathbf{\Theta}_{k-1}\|^2_{\mathbb{C}^D} + \upsilon\|\mathcal{I}_k(\zeta_k)\|^2_{\mathbb{C}^D} \tag{6}$$

where υ is the Lagrange multiplier. The precoding index ζ_k is updated by using stochastic subgradient descent method as [29]

$$\begin{aligned}\zeta_{k+1} &= \zeta_k - \mu\frac{\partial\mathscr{L}(\zeta_k)}{\partial\zeta_k}\\ &= \zeta_k + \mu\langle\boldsymbol{\mathcal{I}}_k(\zeta_k) \odot \boldsymbol{\mathcal{I}}_k'(\zeta_k), \mathbf{U}\hat{\mathbf{\Sigma}}^{-\zeta_k}\Phi\mathbf{r}_k\rangle\end{aligned} \tag{7}$$

where μ is the step size for update of ζ_k, and $\boldsymbol{\mathcal{I}}_k'(\zeta_k) = \frac{\partial\boldsymbol{\mathcal{I}}_k'(\zeta_k)}{\partial\mathbf{\Delta}_k}$. $\hat{\Sigma}$ is obtained by taking truncated SVD of $\mathbf{H}$ [6], and $\mathbf{\Phi}_{p,q} = \log(\mathbf{\Sigma}_{p,q})$ for $p = q$ and 0 for $p \neq q$. Since received signal is impaired by nonlinearity characteristics of LED $f(.)$, therefore the inner product in (7) is computed in a finite dimensional Euclidean space $\mathbf{C}^D$ by using a random Fourier feature (RFF) map $\Psi_\Omega : \mathbb{C}^d \longrightarrow \mathbb{C}^D$ (where $D >> d$), which transforms a nonlinearly separable problem in input space $\mathbb{C}^d$ to a linear problem in finite/high dimensional space $\mathbb{C}^D$. The final update equation for ζ_k can be written as

$$\zeta_{k+1} = \zeta_k + \mu\mathbb{P}\Big(\langle\Psi_\Omega(\boldsymbol{\mathcal{I}}_k(\zeta_k)\odot\boldsymbol{\mathcal{I}}_k'(\zeta_k)), \Psi_\Omega(\mathbf{U}\hat{\Sigma}^{-\zeta_k}\Phi\mathbf{r}_k)\rangle_{\mathbb{C}^D}\Big) \tag{8}$$

where $\mathbb{P}(\cdot)$ is the projector operator in order to ensure $\zeta_k \in [0, 1]$ [5], and RFF map $\Psi_\Omega(\cdot)$ is detailed in the next subsection. Updated ζ_k is also informed to the precoder via a feedback path through RF uplink or transmission at other wavelength in VLC [5], [30].

B. Proposed RFF-KLMS based Post-distortion Algorithm for m-MIMO VLC

In this subsection, RFF-KLMS based post-distorter is proposed for mitigating LED's nonlinear distortions at the receiver. Conventional KLMS with NC based sparsification (KLMS-NC) based post-distorter in RKHS [26] depends on growing dictionary of observations, and it is hard to estimate the memory budget of the dictionary in advance. Hence, RFF is the viable method which approximates the feature map in RKHS by using randomized feature mapping $\Psi_\Omega : \mathbb{R}^d \longrightarrow \mathbb{R}^D$ defined as [22], [31]:

$$\Psi_\Omega(\mathbf{x}) = \sqrt{\frac{2}{D}} \begin{bmatrix} \cos(\boldsymbol{\omega}_1^T \mathbf{x} + \gamma_1) \\ \cos(\boldsymbol{\omega}_2^T \mathbf{x} + \gamma_2) \\ \vdots \\ \cos(\boldsymbol{\omega}_D^T \mathbf{x} + \gamma_D) \end{bmatrix} \tag{9}$$

where each $\{\boldsymbol{\omega}_i\}_{i=1}^D$ is drawn from Fourier transform of the Gaussian kernel $K_{\text{G}}(\boldsymbol{\omega})$ defined as [22]

$$K_{\text{G}}(\boldsymbol{\omega}) = \left(\frac{\sigma}{2\pi}\right)^D e^{(-0.5\sigma^2 ||\boldsymbol{\omega}||^2)} \tag{10}$$

which is the expression for Gaussian distribution $\mathcal{N}(\mathbf{0}_D, \frac{1}{\sigma^2}\mathbf{I}_D)$, each $\{\gamma_i\}_{i=1}^D$ is generated from a uniform probability density function (PDF) in the range $[0 \;\; 2\pi]$ [22], and $\boldsymbol{\Omega} \in \mathbb{R}^{(d+1)\times D}$ is given as

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\omega}_1 & \boldsymbol{\omega}_2 & \dots & \boldsymbol{\omega}_D \\ \gamma_1 & \gamma_2 & \dots & \gamma_D \end{bmatrix} \tag{11}$$

Using RFF, the update equation for filter coefficients $\boldsymbol{\Theta}_k$ for the proposed RFF-KLMS algorithm can be written as [22]

$$\begin{aligned} \boldsymbol{\Theta}_{k+1} &= \boldsymbol{\Theta}_k + \eta_{\text{RFF-KLMS}} \mathbf{e}_k \Psi_\Omega^T(\mathbf{r}'_k) \\ &= \eta_{\text{RFF-KLMS}} \sum_{i=1}^{k-1} \mathbf{e}_i \Psi_\Omega^T(\mathbf{r}'_i) \end{aligned} \tag{12}$$

where $\eta_{\text{RFF-KLMS}}$ is step-size for the proposed RFF-KLMS post-distorter, and $\mathbf{e}_k = \mathbf{s}_k - \boldsymbol{\Theta}_k^T \Psi_\Omega(\mathbf{r}'_k)$ is the error vector. Hence, output of the proposed RFF-KLMS algorithm can be written as

$$\hat{\mathbf{s}}_k = \boldsymbol{\Theta}_k^T \Psi_\Omega(\mathbf{r}'_k) = \eta_{\text{RFF-KLMS}} \sum_{i=1}^{k-1} \mathbf{e}_i \langle \Psi_\Omega^T(\mathbf{r}'_i), \Psi_\Omega^T(\mathbf{r}'_k) \rangle_{\mathbb{C}^D} \tag{13}$$

Hence, it can be observed from (13) that output of the proposed RFF-KLMS based post-distorter has a fixed size solution instead of growing kernel functions, which performs the post-distortion under fixed memory budget constrained scenarios as in a practical system. The pseudo-code of the proposed AP-RFF-KLMS algorithm is given in **Algorithm 1**.

Computational complexity: The computational complexity of RFF based algorithms depends upon the number of RFF dimensions D [22] while the computational complexity of dictionary based algorithms in RKHS depends upon the cardinality of dictionary (denoted by $|\mathscr{D}_k|$). Hence, the overall computational complexity[1] of the proposed AP-RFF-KLMS, and AP-KLMS-NC algorithm is $O(MD)$, and $O(M|\mathscr{D}_k|)$ [22], respectively, where M is the order of m-MIMO VLC channel matrix.

Algorithm 1 Proposed AP-RFF-KLMS Algorithm for m-MIMO VLC

Input: Received symbols $\mathbf{r}_k$

Initialization:

- $k = 1$, Maximum iterations: MAXITER $\eta_{\text{RFF-KLMS}}$, μ, $\boldsymbol{\Theta}_1 = \mathbf{0}$, Kernel width σ, and RFF Dimension D
- Generate iid $\{\boldsymbol{\omega}_i\}_{i=1}^D$ from Gaussian distribution $\mathcal{N}(0, \frac{1}{\sigma^2}\mathbf{I}_D)$ given in (10).
- Generate iid $\{\gamma_i\}_{i=1}^D$ from uniform distribution in the range $[0, \; 2\pi]$.

Computation:

while $k \leq$ MAXITER **do**

    Compute RFF vector: $\Psi_\Omega(\mathbf{r}'_k)$ via (9).
    Compute post-distorter output: $\hat{\mathbf{s}}_k = \boldsymbol{\Theta}_k^T \Psi_\Omega(\mathbf{r}'_k)$
    Update ζ_k via (8)
    Compute error, $\mathbf{e}_k = \mathbf{s}_k - \hat{\mathbf{s}}_k$.
    Adapt $\boldsymbol{\Theta}_{k+1} = \boldsymbol{\Theta}_k + \eta_{\text{RFF-KLMS}} \mathbf{e}_k \Psi_\Omega^T(\mathbf{r}'_k)$

end while

Output: Estimated symbols $\hat{\mathbf{s}}_k$

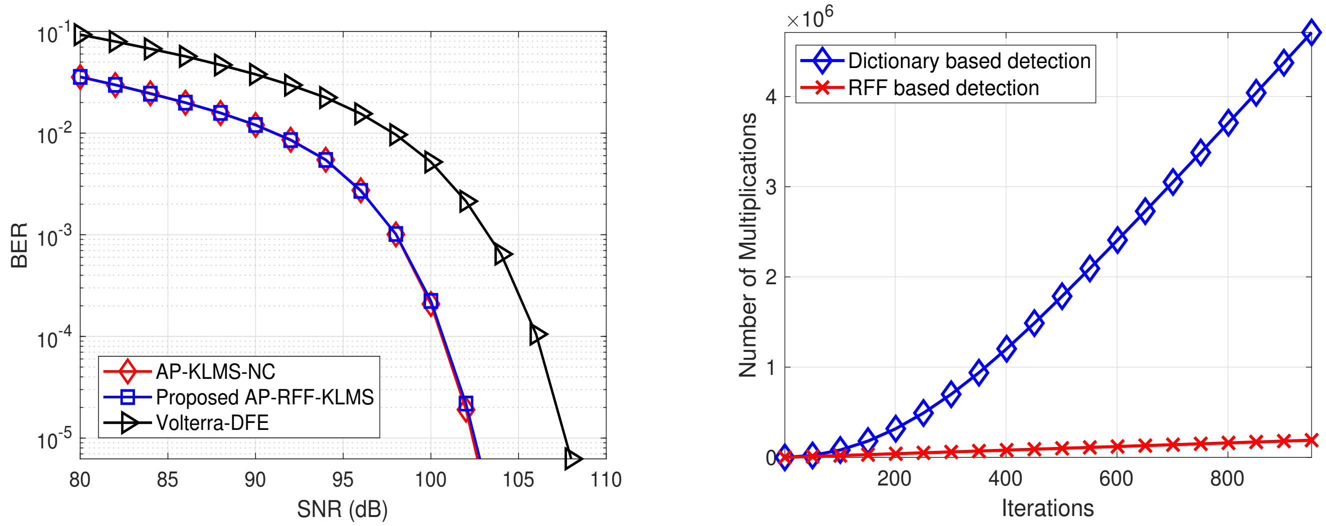
IV. Simulations

This section presents simulation results for the proposed AP-RFF-KLMS algorithm. 16×16 and 36×36 indoor m-MIMO VLC channel is considered [4] with the channel specifications given in Table 1. 16-CAP-QAM modulation scheme is considered in all the simulations. 100 independent Monte-Carlo simulations are performed over 10^6 symbols. Parameters for Rapp's model are as follows: $V_{\text{max}} = 0.5$ V, and $q = 0.5$ [19]. Performance of the proposed AP-RFF-KLMS algorithm is compared with the following conventional scenarios: (1) Volterra-DFE, and (2) AP-KLMS-NC (dictionary based post-distorter in RKHS). Conventional Volterra-DFE algorithm with 55 first order and 35 second order taps is considered. Silverman's rule is adopted to choose kernel width σ, step-size is chosen as 0.35, and sparsification thresholds $\delta_o = 10^{-3}$, $\delta_e = 0.15$ [6], [17] are chosen for the AP-KLMS-NC algorithm. Simulation parameters for the proposed AP-RFF-KLMS are chosen as follows: $\upsilon = 1.5$, $\beta = 5$, $\mu = 0.077$ and $\zeta_k = 1$ for $k = 1$. kernel parameter $\sigma = 0.85$, $\eta_{\text{RFF-KLMS}} = 0.35$, and RFF dimension is chosen as $D = 250$ for 36×36 and $D = 200$ for 16×16 m-MIMO channel (i.e., $D > 2d$, where d is the dimension of input space [22]).

The plots for SNR vs BER[2], and evolution of computational complexity with the number of iterations are shown in Fig.

[1]It is noteworthy that the computational complexity of the proposed adaptive precoder (using truncated SVD) is $O(rM)$ instead of $O(M^2)$ for full SVD based adaptive precoder [5]. Since r is very small due to correlatedness in the columns of the channel matrix, therefore the overall computational complexity of the proposed AP-RFF-KLMS algorithm is $O((r + D)M)$, which is approximately equal to $O(MD)$.

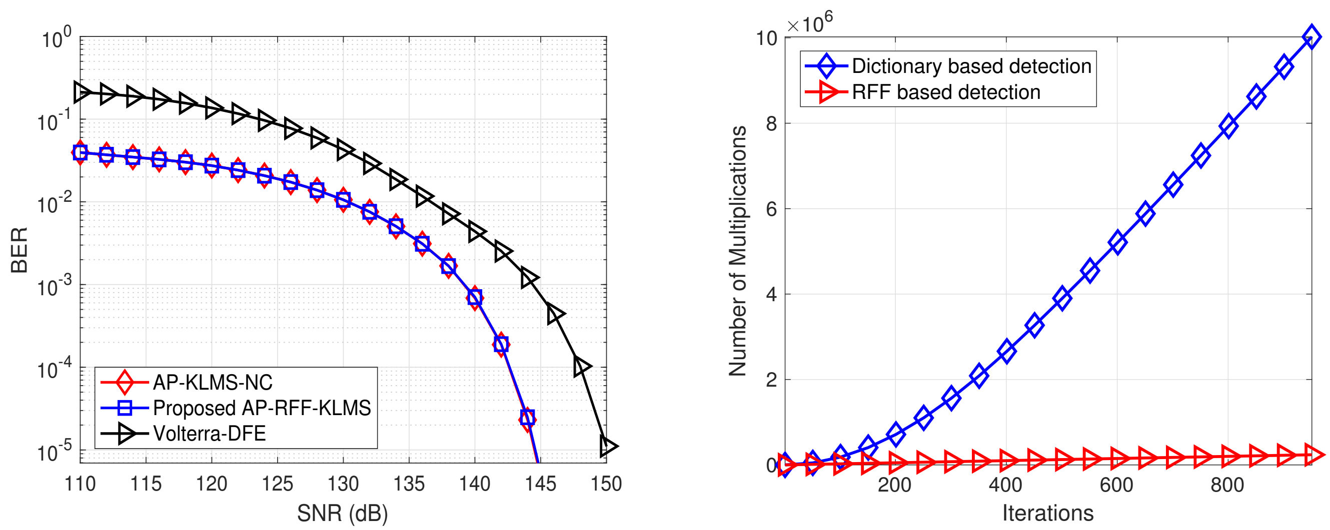
[2]It is important to note that an array of LEDs provide high illumination (i.e., levels of 400-800 lux) which ensures that high SNR levels are practically achievable for m-MIMO VLC systems [3]. Since m-MIMO VLC channel matrices are ill-conditioned, therefore, the waterfall region of the BER vs SNR characteristics is achieved at SNRs (of the order of hundreds of dBs as in [32], [33]) that are significantly higher than analogous RF communication systems [32], [33].



(a) SNR vs BER performance

(b) Computational complexity vs number of iterations

Fig. 2. BER performance and computational complexity for the proposed AP-RFF-KLMS algorithm for 16×16 m-MIMO VLC



(a) SNR vs BER performance

(b) Computational complexity vs number of iterations

Fig. 3. BER performance and computational complexity for the proposed AP-RFF-KLMS algorithm for 36×36 m-MIMO VLC

2 for 16×16 m-MIMO VLC channel. As inferred from Fig. 2, the proposed AP-RFF-KLMS algorithm delivers 5 dB gain at BER of 10^{-5} over the conventional Volterra-DFE algorithm. Furthermore, it can be observed from Fig. 2 that the proposed AP-RFF-KLMS algorithm has similar BER performance with huge computational savings (in terms of number of multiplications) as compared to dictionary based AP-KLMS-NC algorithm.

Next, SNR vs BER performance and computational complexity vs iterations are presented in Fig. 3 for 36×36 m-MIMO VLC channel. As inferred from Fig. 3 that the proposed AP-RFF-KLMS algorithm delivers 5 dB gain at BER of 10^{-5} over the conventional Volterra-DFE algorithm. It can be observed from Fig. 3 that computational complexity of the proposed AP-RFF-KLMS algorithm is lower with similar BER performance as compared to AP-KLMS-NC algorithm.

V. Conclusion

In this paper, a fixed budget adaptive precoder and RFF-KLMS based post-distorter is proposed to reduce the condition number of the overall m-MIMO VLC channel, and to mitigate LED nonlinearity. An MSER based optimization problem is formulated in a finite dimensional Euclidean space for optimization of precoding exponent in order to compromise between condition number and PAPR. Simulations performed over m-MIMO VLC channels indicate that the proposed AP-RFF-KLMS based post-distorter delivers similar BER performance with lower computational complexity as compared to

TABLE I
SIMULATION PARAMETERS FOR M-MIMO VLC CHANNEL [4]

PARAMETERS	SPECIFICATIONS
Distance between two neighbour LEDs	5.8 cm
Distance between two neighbour PDs	5.8 cm
Effective aperture area of detector	1 cm^2
Field of view (FOV)	$10°$
Transmitter semi-angle at half power ($\phi_{\frac{1}{2}}$)	$10°$
Distance between transmitter and receiver plane	2 m
Centre of LED array	(11.6 cm, 11.6 cm, 0.5 m)
Centre of PD array for $N_t = N_r = 36$	(14.3 cm, 15.7 cm, 2.1 m)
Centre of PD array for $N_t = N_r = 16$	(15.6 cm, 16.7 cm, 1.5 m)

the dictionary based KLMS-NC algorithm in RKHS. Hence, the proposed RFF based post-distortion algorithm have a finite memory budget, which makes it viable for practical deployments of m-MIMO VLC systems for beyond 5G and 6G communication systems.

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