Comparative Study of Adaptive Beamforming Algorithms for Smart antenna Applications

Mohammad Abualhayja'a
Electrical Engineering Department
United Arab Emirates University
Al Ain, UAE
201770327@uaeu.ac.ae

Mousa Hussein
Electrical Engineering Department
United Arab Emirates University
Al Ain, UAE
mihussein@uaeu.ac.ae

***Abstract*— Beamforming is signal processing techniques used to shape the antenna array pattern according to prescribed criteria. In this paper, a comparative study is presented for various adaptive antenna beamforming algorithms. Least Mean Square (LMS), Normalized Least Mean Square (NLMS), Recursive Least Square (RLS) and Sample Matrix Inversion (SMI) algorithms are studied and analyzed. we also consider some possible adaptive filters combinations, such as LMS with SMI weights initialization, and combined NLMS filters with a variable mixing parameter. These algorithms are simulated for a linear antenna array with different sizes, and results are discussed in terms of their Convergence speed, Max SLL and Null depths.**

Keywords*— *Adaptive beamforming, LMS algorithm, NLMS algorithm, RLS algorithm, SMI algorithm.

I. Introduction

As a key technology for modern wireless communication systems, especially 5G networks, adaptive beamforming became an intense field of study. The tendency of involving much higher frequencies and higher order modulation increased the demand for maximizing the power utilization which can be achieved by focusing the RF resources where they are most needed. At the same time, we can eliminate any source of interference or improve the signal to interference noise ratio (SINR), this can be achieved using adaptive beamforming techniques [1-3]. Adaptive beamforming is a signal processing approach that spatially filters the antenna array input by steering the antenna beam toward the desired signal and forming nulls at the directions of interference [4].

Least Mean Square (LMS), Normalized Least Mean Square (NLMS), Recursive Least Square (RLS) and Sample Matrix Inversion (SMI) algorithms are widely used for antenna array adaptive beamforming, these algorithms can be characterized in terms of their convergence properties and computational complexity [5-8]. In this paper, we first discuss these algorithms, followed by implementing possible combinations of those algorithm, such as LMS with SMI weights initialization, and combination of two NLMS filters algorithm with a variable mixing parameter.

II. System Modeling

As shown in Fig. 1, consider a model of a linear antenna array which is composed of N uniformly distributed isotropic antenna elements, the output of the array is given by the following equation [5]:

$$y(k) = w^H(k).x(k) \quad (1)$$

where $w(k)$ is the array weight vector and $x(k)$ is the received signal vectors.

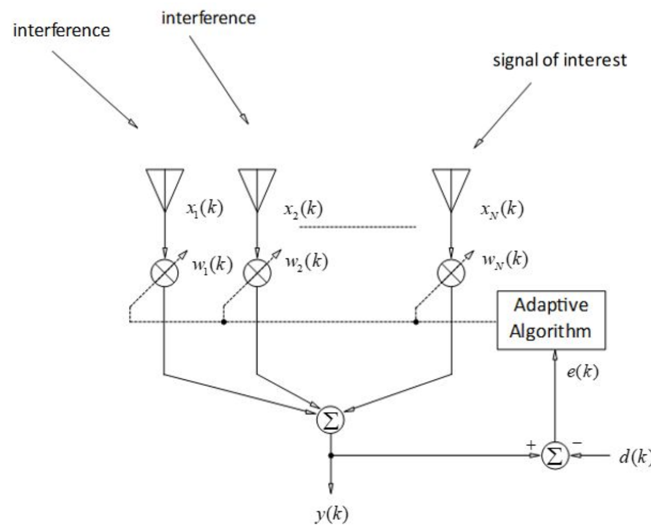

Fig.1. Adaptive antenna array system.

The problem that adaptive beamforming addresses is how to adjust the array weights in order to drive the array output $y(k)$ to the desired output $d(k)$, accordingly we may define an estimation error $e(k)$:

$$e(k) = d(k) - w^H.x(k) \quad (2)$$

Adaptive algorithms are basically used to minimizes the resulting error statistically, which is to solve [6]:

$$\text{Min}E[e(k)e^*(k)] \quad (3)$$

with $E[.]$ representing the expectation operator.

III. Adaptive Beamforming Algorithms

A. Least Mean Square (LMS) Algorithm

The LMS algorithm is a stochastic gradient algorithm. Its computational simplicity, easy coding and robustness, are

978-1-7281-6535-6/20/$31.00 ©2020 IEEE

significant features make it one of the most used adaptive filtering algorithms [6].

Given the received signal vector, $x(k)$, and the desired output, $d(k)$, the LMS algorithm updates filter taps or array weights iteratively, using the following equation [5]:

$$w(k+1) = w(k) + \mu x(k) e^*(k) \tag{4}$$

where μ is the LMS algorithm step size.

It turns out that convergence of the filter is related to the step size, that is μ should satisfy the following condition, $0 < \mu < 2/\lambda_{max}$ where λ_{max} is the maximum eigenvalue of the input vector autocorrelation matrix.

B. Normalized Least Mean Square (NLMS) Algorithm

The LMS algorithm weights update is driven by the input vector $x(k)$, which raises the probability of having a gradient noise amplification problem in case of large $x(k)$ values. Moreover, the convergence of the LMS algorithm is relatively slow, hence, the normalized least mean square (NLMS) algorithm is proposed to overcome the gradient noise amplification problem and more importantly significantly increase the convergence rates. As shown in (5), compared to the LMS algorithm, the step size of the NLMS algorithm is time varying, since the weights corrector term is normalized with respect to the norm of input vector weights update.

$$w(k+1) = w(k) + \frac{\mu_{NLMS}}{\|x(k)\|^2 + \alpha} x(k) e^*(k) \tag{5}$$

where μ_{NLMS} is the NLMS adaptation constant and α is a small positive constant to avoid division by zero.

C. Recursive Least Square (RLS) Algorithm

In the method of least squares, we can find the optimum filter taps that minimizes the estimated error by projecting the desired output vector on the column space of the input sequence matrix using the modified weighting vector:

$$w^*(k) = A^{\dagger} d(k) \tag{6}$$

where $A^{\dagger} = (A^T A)^{-1} A^T$ is the pseudoinverse of the input sequence matrix A.

The RLS algorithm is developed from the method of least squares using the matrix inversion lemma, so we can obtain the updated weights vector $w(k)$ from the old-squares estimate $w(k-1)$ without performing any matrix inversion calculations by utilizing the input vector sequence. The RLS algorithm is firstly initiated by setting the weights vector $w(k)$ and the correlation matrix inverse $P(k)$ as follows [5-6]:

$w(0) = 0$, and $P(0) = \delta^{-1} I$, where δ is a small positive constant.

The weights vector and the correlation matrix inverse are updated as follows:

$$w(k) = w(k-1) + g(k)\xi^*(k) \tag{7}$$

$$P(k) = \lambda^{-1} P(k-1) - \lambda^{-1} g(k) x^H(k) P(k-1) \tag{8}$$

where $\xi(k) = d(k) - w^H(k-1)x(k)$ is the prior estimated error and $g(k)$ is the gain vector which is defined as:

$$g(k) = \frac{\lambda^{-1} P(k-1) x(k)}{1 + \lambda^{-1} x^H(k) P(k-1) x(k)} \tag{9}$$

where λ is the forgetting factor, a positive constant less than 1.

Although the RLS rate of convergence is faster than the LMS algorithm, but in terms of calculations complexity the RLS algorithm is significantly costly.

D. Sample Matrix Inversion (SMI) Algorithm

In the LMS algorithm system goes through many iterations to drive the output toward the desired signal, and in case of rapidly changing signal characteristics, system may not approach an acceptable convergence. A solution to this is to calculate the time average estimate of the correlation matrix by using K-length block of data, this approach is called sample matrix inversion (SMI).

By dividing the input data into k blocks, we define the array correlation matrix as the following [7-9]:

$$R_{xx} = \overline{X}_K(k) \overline{X}_K^H(k) \tag{10}$$

where $\overline{X}_K(k)$ is the k^{th} block of input vector ranging over K samples of data.

$$\overline{X}_K(k) = \begin{bmatrix} x_1(1+kK) & x_1(2+kK) & \dots & x_1(K+kK) \\ x_2(1+kK) & x_2(2+kK) & & \vdots \\ \vdots & & \ddots & \\ x_M(1+kK) & \dots & & x_M(K+kK) \end{bmatrix} \tag{11}$$

We can also define the desired output vector by:

$$d(k) = [d(1+kK) \quad d(2+kK) \quad \dots \quad d(K+kK)] \tag{12}$$

and the estimate of correlation vector by:

$$\overline{p}(k) = \tfrac{1}{K} d^*(k) \overline{X}_K(k) \tag{13}$$

The weights vector update equation is given by:

$$w(k) = \overline{R}_{xx}^{\;-1}(k) \overline{p}(k) \tag{14}$$

One drawback of the SMI algorithm is that it is not sufficient for large number of antenna elements, but it can be used for weights initialization when combined with other algorithms as in [9].

E. Combination of two NLMS filters with variable mixing parameter

As noted for both LMS and NLMS algorithms, convergence rates depend on the step size. On the other hand, there is a tradeoff between convergence speed and the ability of tracking the desired signal in a satisfactory manner.

In order to increase convergence rate and ensure system robustness, many combined adaptive filters are proposed using an adaptive mixing parameter $\lambda(k)$ [11-13].

Consider a system of two combined NLMS filters with different adaptation constants, μ_1 and μ_2, with $\mu_1 > \mu_2$, then the output $y(k)$ is given by:

$$y(k) = \lambda(k)y_1(k) - (1 - \lambda(k))y_2(k) \qquad (15)$$

As proposed in [12], $\lambda(k)$ is constrained to the interval [0,1] using an auxiliary variable $\alpha(k)$, where $\lambda(k) = \frac{1}{(1+e^{-\alpha(k)})}$, and $\alpha(k)$ is updated as follows:

$$\alpha(k+1) =$$
$$\alpha(k) + \mu_\alpha e(k)(y_1(k) - y_2(k))\lambda(k)(1 - \lambda(k)) \qquad (16)$$

To ensure a continuous adaptation of the mixing parameter, $\alpha(k)$ is limited between $[-\alpha+, \alpha+]$ [12],[13].

The combined weight vector is defined as:

$$w(k) = \lambda(k)w_1(k) - [1 - \lambda(k)]w_2(k) \qquad (17)$$

and each filter updates its weight vector using the NLMS algorithm:

$$w_i(k+1) = w_i(k) + \frac{\mu_i}{\|x(k)\|^2 + \alpha} x(k)e_i^*(k) \qquad (18)$$

I. SIMULATION RESULTS

In this section, a linear array is used to evaluate each algorithm with different number of elements. The array receives five narrowband signals, a desired signal and four interference signals from the azimuth of 35, 50, 10, -30 and -45 respectively. The signal to interference noise ratio (SINR) is 30 dB and the spacing between array elements is set to be $\lambda/2$.

The step size for both LMS and NLMS are 3×10^{-3} and 1.2 respectively, and the RLS forgetting factor is 0.9 and δ is 0.01, whereas the combined NLMS parameters are $\mu_1 = 0.9$, $\mu_2 = 1.7$ and $\mu_\alpha = 1$.

Figures 2 to 4 show the normalized array gain for the LMS, NLMS, RLS, SMI, LMS/SMI and combined NLMS algorithms respectively, using 8, 16 and 21 elements. It can be observed that the RLS, SMI and LMS with SMI weights initialization show deep nulls, on the other hand, they have the highest SLL. Whereas LMS, NLMS and combined NLMS introduce the lowest SLL, where both NLMS and combined NLMS have deeper null than LMS. However, the LMS shows better performance when using higher number of elements.

Figures 5 to 8 show the resulting MSE versus iterations for the LMS, NLMS, RLS, LMS/SMI and combined NLMS algorithms with different array sizes, and Fig.9 shows the MSE of the SMI algorithm for each block of data. The SMI, RLS, and LMS/SMI algorithms have faster convergence rates compared to the LMS, NLMS, and combined NLMS algorithms. Again, the performance of LMS shows improvement when using higher number of elements, whereas SMI gives higher MSE with larger array size. Compared to the LMS algorithm, the LMS/SMI algorithm MSE steps to the optimum value due to SMI weights initialization, whereas in the combined NLMS algorithm, an improved convergence rates are achieved compared to the NLMS algorithm.

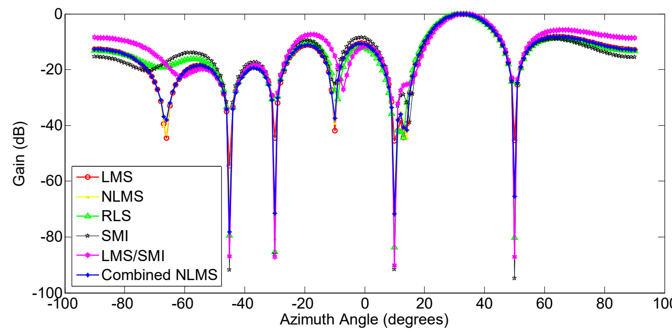

Fig.2: Normalized gain for 8 elements antenna array using different adaptive algorithms.

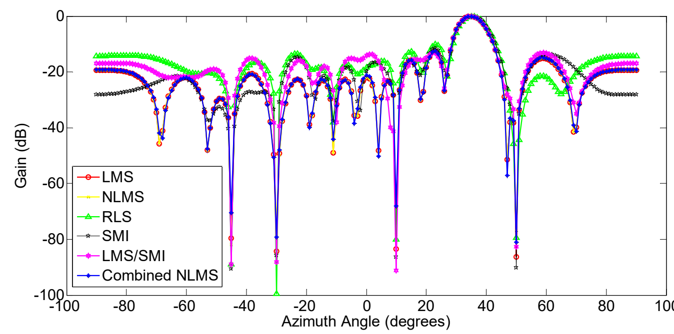

Fig.3: Normalized gain for 16 elements antenna array using different adaptive algorithms.

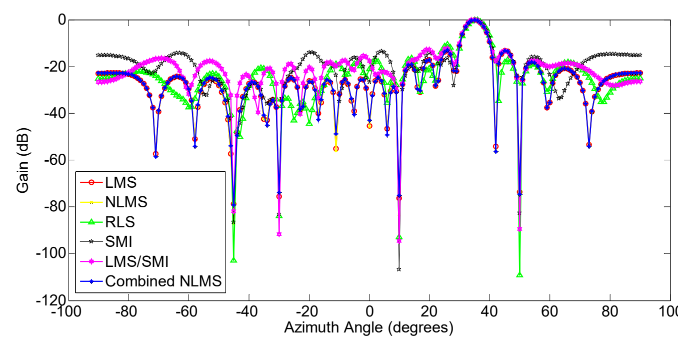

Fig.4: Normalized gain for 21 elements antenna array using different adaptive algorithms.

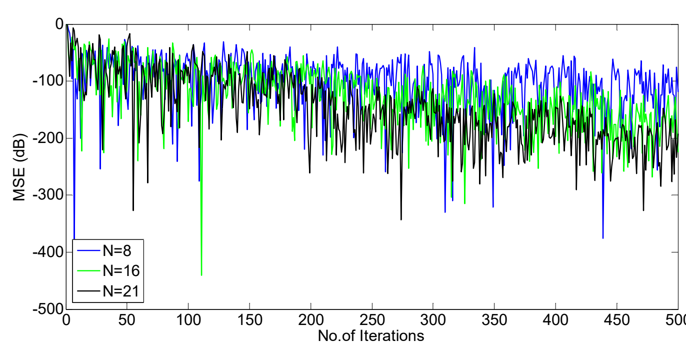

Fig.5: MSE versus iterations for LMS algorithm with different number of array elements.

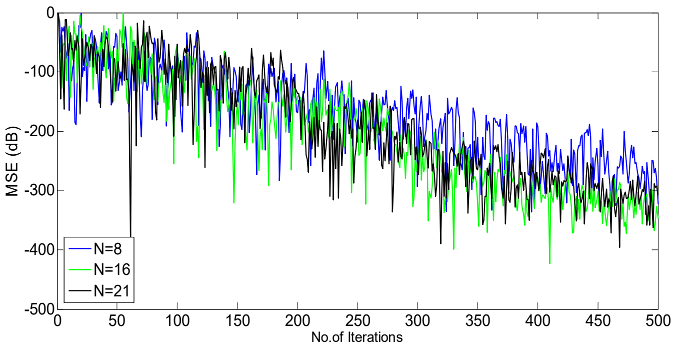

Fig.6: MSE versus iterations for NLMS algorithm with different number of array elements.

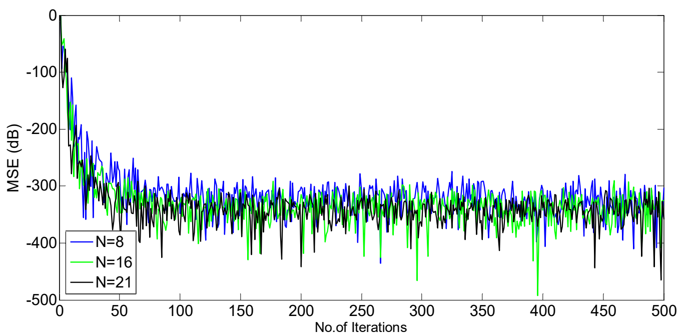

Fig.7: MSE versus iterations for RLS algorithm with different number of array elements.

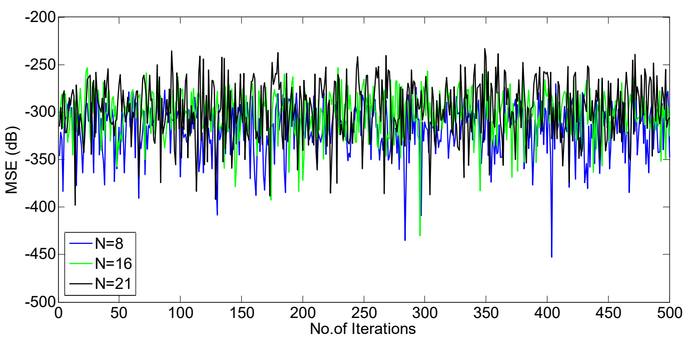

Fig.8: MSE versus iterations for LMS/SMI algorithm with different number of array elements.

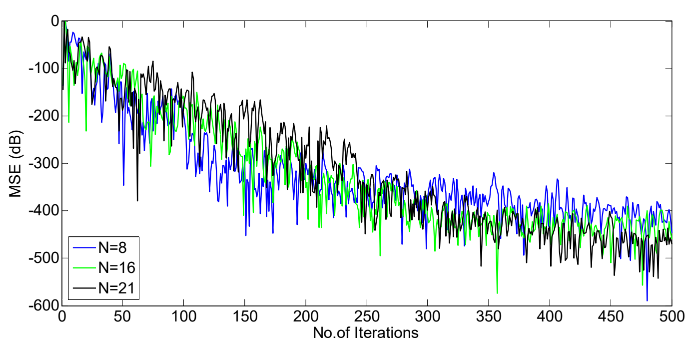

Fig.8: MSE versus iterations for Combined NLMS algorithm with different number of array elements.

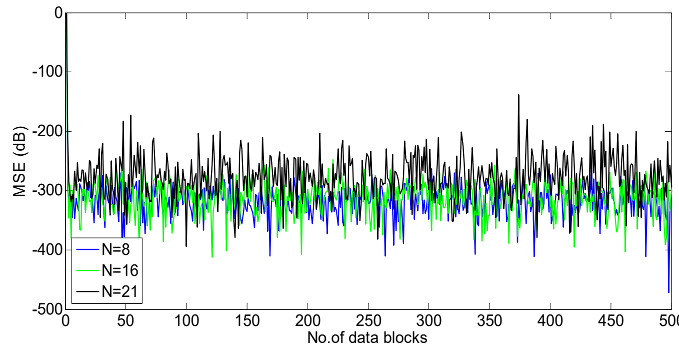

Fig.9: MSE versus iterations for SMI algorithm with different number of array elements.

II. CONCLUSION

In this paper, we presented and analyzed various beam forming algorithms such as: Least Mean Square (LMS), Normalized Least Mean Square (NLMS), Recursive Least Square (RLS), Sample Matrix Inversion (SMI), LMS with SMI weights initialization, and combined NLMS filter with a variable mixing parameter. Simulation results for a linear array, show that each algorithm has advantages and weaknesses. In the terms of convergence speed and nulls depth RLS and SMI show better performance, whereas LMS, NLMS are simple and give lower SSL. However, it can be observed that some of these weaknesses can be reduced by using combined algorithms, where LMS/SMI and combined NLMS filters have an improved convergence speed compared to the LMS and NLMS algorithms with an acceptable increase in the computation complexity.

ACKNOWLEDGMENT

The authors wish to acknowledge the support provided by United Arab Emirates University.

REFERENCES

[1] N. Al-Falahy, and O. Alani. “Technologies for 5G Networks: Challenges and Opportunities,” IT Professional, January 2017, 19(1), pp.12-20.

[2] Y. Niu, Y. Li, , D. Jin, , L. Su, and A. Vasilakos, “A survey of millimeter wave communications (mmWave) for 5G: opportunities and challenges”, Wireless Networks, April 2015, 21(8), pp.2657-2676.

[3] C. A. Balanis, “Antenna theory: analysis and design,” 3rd ed., John Wiley & Sons, 2005.

[4] B. D. Van Veen, and K. M. Buckley. “Beamforming: A versatile approach to spatial filtering,” ASSP Magazine, IEEE 5, no. 2 (1988): 4-24.

[5] C. Balanis, and P. Ioannides. “Introduction to smart antennas”, [San Rafael, CA]: Morgan and Claypool Publishers, 2007.

[6] S. Haykin, Adaptive Filter Theory, 4th ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.

[7] D. N. Patel, B. J. Makwana, and P. B. Parmar, “Comparative analysis of adaptive beamforming algorithm LMS, SMI and RLS for ULA smart antenna,” International Conference on Communication and Signal Processing (ICCSP), April 2016, doi: 10.1109/iccsp.2016.7754305

[8] P. Saxena, and A. Kothari. “Performance Analysis of Adaptive Beamforming Algorithms for Smart Antennas,” IERI Procedia, 2014, 10, pp.131-137.

[9] R. Shubair, S. Jimaa, and A. Omar. “ Enhanced adaptive beamforming using LMMN algorithm with SMI initialization”. 2009 IEEE Antennas and Propagation Society International Symposium, July 2009, doi: 10.1109/aps.2009.5171469

[10] H. L. Van, Trees, “Detection, Estimation, and Modulation Theory, Part IV: Optimum Array Processing,” John Wiley & Sons, New York, NY, 2002.

[11] L. Lu, H. Zhao, Z. He, and B. Chen, “A novel sign adaptation scheme for convex combination of two adaptive filters,” AEU - International Journal of Electronics and Communications, July 2015, 69(11), pp.1590-1598.

[12] F. Huang, J. Zhang, and S. Zhang, “Combined-Step-Size Affine Projection Sign Algorithm for Robust Adaptive Filtering in Impulsive Interference Environments,” IEEE Transactions on Circuits and Systems II: Express Briefs, 63(5), May 2016, pp.493-497.

[13] R. Candido, M. Silva, and V. Nascimento. “Transient and Steady-State Analysis of the Affine Combination of Two Adaptive Filters,” IEEE Transactions on Signal Processing, 58(8), March 2010, pp.4064-4078.

[14] F. B. Gross, “Smart Antenna for Wireless Communication,” McGraw-Hill, September 14, 2005.