Network Coding Schemes for Time Variant/Invariant Channels with Smart Acknowledgment

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***Abstract*—In this paper, we propose models and schemes for coded and uncoded packet transmission over time invariant (TIC) and time variant (TVC) channels. We provide an approximation of the delay induced assuming finite number of time slots to transmit a given number of packets. We propose an adaptive physical layer (PHY)-aware coded scheme that designs smart acknowledgments (ACK) via an optimal selection of coded packets to transmit at a given SNR. We apply our proposed schemes to channels with complex fading behavior and high round trip (RTT) delays. We compare the accuracy of TVC coded scheme to the TIC coded scheme, and we show the throughput-delay efficacy of adaptive coded schemes driven by PHY-awareness in the mitigation of high RTT environments, with up to 3 fold gains.**

I. INTRODUCTION

The proposal of network coding (NC) schemes for wireless networks need to take into consideration different aspects in the wireless medium like noise, interference, and fading. It has been shown in [1] that network coding can mitigate the wireless fading via its underlying coding across the packets which increases the diversity.

Moreover, from a decodability perspective, the authors in [2] proposed interference cancellation techniques that requires a precise estimation of channel coefficients for each packet involved in a collision. In [3] an opportunistic network coding approach was introduced. In particular, the codewords are adapted according to the received information from the neighbors. In [4], the authors show that fixed network codes without channel state information (CSI) cannot achieve instantaneous min-cut. However, they proved that adaptive network codes with one bit global CSI have lower erasure probability than the codes without CSI.

Network codes adaptation strategies that are based on channel state information are limited to packet erasure channel model, that is, a two-state Markov model of Gilbert-Elliott channel, in which a packet is whether dropped with a certain probability or received without error, see [5].However, in fading channels, the packet erasures become time dependent, to which adaptive network coding for time variant channels was first proposed in [6], followed by the proposal of adaptive network coding and modulation schemes that are rate and energy efficient, see [7], [8], respectively. In [6] and its following works, PHY-aware adaptive coded transmission schemes that outperform non-adaptive coded schemes and selective repeat ARQ over TVC were proposed. However, the benefits of PHY-awareness in terms of other design parameters, particularly, the ACK packets, were not considered. The assumption of fixed one time slot ACK become different here, where the proposed schemes in this paper considers adaptive number of acknowledgments based on the designed on the fly adaptive coded packets. Therefore, in this paper, we extend the results in [6] to schemes that capitalize on the predictive awareness of optimal number of coded packets to transmit, due to PHY-awareness, that allows the proposal of a coded scheme for TIC/TVC with smart acknowledgment, utilizing better the resources used to ACK packets and avoiding delays when RTT is high.

The paper contributions are three fold:

First, we propose models of uncoded and coded packet transmission over fixed erasure TIC and time dependent erasure TVC channels.

Second, we propose a PHY-aware adaptive coded transmission scheme that measures the optimal number of coded packets by adapting to the predicted channel erasures.

Third, we capitalize on the proposed adaptive scheme to propose and integrate a smart acknowledgment scheme based on the predicted optimal number of coded packets.

We establish a comparison between the network coding scheme for TIC to the one for TVC, and its implication on the adaptive scheme proposed. Additionally, we show around a 3 fold gain in terms of throughput and delay when smart acknowledgment scheme is used to mitigate the large RTT (e.g., satellite communications).

II. CHANNEL MODEL

In a downlink transmission over a wireless channel, the dynamics of rain fading[1] are generally described by

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[1] Rain fading refers to the absorption of a microwave radio frequency signals by atmospheric effects and to the losses encountered at frequencies above 11 GHz. Rain fading is a main issue in Ka-band satellite communication systems, terrestrial point to point microwave systems, or future mmWave cellular systems.

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an auto-regressive moving-average model with order p [9], [10], [11] as follows:

$$h(t) = -\sum_{i=1}^{p} a_i h(t-i) + \omega(t), \tag{1}$$

$\omega(t)$ is a zero mean unit variance white Gaussian process, and the AR correlation coefficient bounded as $0 \leq a_i \leq 1$ corresponding to slow fading at $a_i = 1$ and very fast fading at $a_i = 0$, and we select $p = 1$ (a figure of merit in satellite communications, see [9]).

A. Probability of Error

Assume that there is no channel coding within the received packets. For a packet to be received every symbol need to be received. Therefore, the packet erasure probability at channel state h_j with respect to the bit error probability is given by:

$$P_e(h_j) = 1 - (1 - P_b(h_j))^B \tag{2}$$

B is the total number of bits in a packet. P_b is the bit error probability which is dependent on the channel model, and is considered to be log-normally fading channel. We consider a pre-knowledge of the first channel state h_0. Therefore, for a given channel gain $G_0 = \mathbb{E}[h_0(t)^2]$ that is log-normally distributed, the probability of error is given by:

$$P_b(G_0) = \int_0^{\infty} Q\left(\sqrt{SNRG_0}\right) p_G(G_0), \tag{3}$$

With the one-dimensional Gaussian Q-function defined as:

$$Q\left(x\right) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{u^2}{2}} du, \tag{4}$$

and the channel gain probability density function is given by:

$$p_G(G_0) = \frac{4.3429}{G_0 \sigma \sqrt{2\pi}} e^{-(10\log_{10} G_0 - m)/2\sigma^2}, \tag{5}$$

where $10\log_{10} G(t)$ follows a Gaussian distribution. A typical value is to choose $m = -0.5$, and $\sigma^2 = 1$ [12]. Therefore, with one CSI feedback and pre-knowledge of h_0, we predict the rest of the channel states by the first order auto-regression. The distribution of the new channel gain given the previous $p(h_k(t)|h_{k-1}(t-1))$ is Gaussian with mean $ah_{k-1}(t-1)$, and variance equals to the noise variance which is 1. Therefore, the bit error probability of the predicted states can be given by:

$$P_b(h_k|h_{k-1}) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} Q\left(\sqrt{SNR|h_k|^2}\right) \times p_{h_k|h_{k-1}}(h_k|h_{k-1}) dh_k, \tag{6}$$

and due to the moving average $h_k \sim \mathcal{N}(ah_{k-1}, 1)$, the conditional Gaussian fading channel gain probability is given by:

$$p_{h_k|h_{k-1}}(h_k|h_{k-1}) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{(h_k - ah_{k-1})^2}{2\sigma^2}} du \tag{7}$$

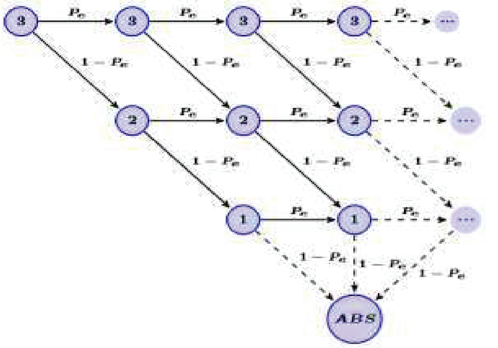

Figure 1. Markov Model for 3 Packets Transmission over TIC

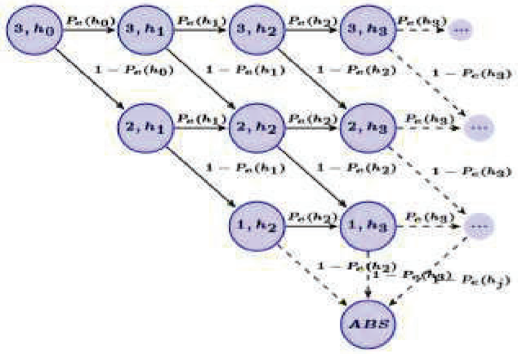

Figure 2. Markov Model of 3 Packets Transmission over TVC

III. MODELS OF CODED AND UNCODED PACKET TRANSMISSION SCHEMES OVER TIC AND TVC

For a TIC, the packet erasures are fixed and time independent. However, due to the channel variation over time, in TVC, the packet erasures become time dependent. Figure 1 illustrates the model of packet transmission over a TIC, and Figure 2 illustrates the model of packet transmission over TVC.

We model an N packet transmission over a slotted time. Therefore, the single packet transmission will occur over one or more number of time slots. Each packet transmission corresponds to a vector of transmitted symbols that suffer from a deterministic or fixed fading coefficients (in TIC) or from e.g., log-normal fading over time (in TVC). Each vector is represented by one state in the Markov chain. We assume a finite number of time slots to transmit N packets. In particular, we assume a finite number of time slots $(\sum(N) + 1)$ per packet transmission, creating a transition matrix P of size equals to $((\sum(N) + 1) \times N) \times ((\sum(N) + 1) \times N)$, required to transmit the packets successfully, and we consider pre-knowledge about the first CSI h_0 the first packet will encounter. Therefore, if the channel is time variant (TVC), a channel coefficient corresponds to one packet transmission. However, the channel is fixed over all packets if the channel is time invariant (TIC).

For instance, we explain the TVC model: In order to transmit 3 packets, first packet transmission at state $(3, h_0)$ will be either successfully delivered at channel state h_0 with probability $1 - P_e(h_0)$. Therefore, the transition will occur to state $(2, h_1)$, or fail to be delivered with probability $P_e(h_0)$ causing a transition to state $(3, h_1)$, and then the transition will occur to states $(3, h_3)$ or $(2, h_2)$ whether the packet fail or delivered successfully, respectively. If the last packet at state $(1, h_k)$, $k \geq 3$ is delivered successfully after the channel evolution over time, i.e., after a certain number of time slots, the chain will be absorbed. The transition matrix P is of size 21×21 based on the assumption of finite number of states to transmit the 3 for delay approximation. If the channel is TIC, the same process applies but with fixed P_e, dropping the channel variation parameters.

A. The expected time to transmit i native (uncoded) packets

The expected time to deliver i uncoded packets can be written as follows,

$$\begin{aligned} T(i, h_j) = T_d(i, h_j) + p_{(i,h_j)\to(i-1,h_{j+1})} T(i-1, h_{j+1}) \\ + p_{(i,h_j)\to(i,h_{j+1})} T(i, h_{j+1}) \end{aligned} \tag{8}$$

The time to deliver a packet at a given channel state equals the packet length $T_d(i, h_j) = T_p$, and the transition probability $p_{(i,h_j)\to(i-1,h_{j+1})}$ is given by:

$$p_{(i,h_j)\to(i-1,h_{j+1})} = 1 - P_e(h_j), \tag{9}$$

and the probability of failure in transmitting one coded or uncoded packet $p_{(i,h_j)\to(i,h_{j+1})}$ is given by:

$$p_{(i,h_j)\to(i,h_{j+1})} = P_e(h_j), \tag{10}$$

where $P_e(h_j)$ is the packet erasure probability when the channel $h(t) = h_j$ for the duration of the packet transmission.

With the introduction of coding across the packets, i.e., network coding, the expected time to deliver N_i coded packets will be a function of the expected time to deliver the coded packets at their given - or predicted - states can be written as follows,

$$T(i, h_j) = T_d(N_i, h_j) + \sum_{l=1}^{i} P^{N_i}_{(i,h_j)\to(l,h_{j+N_i})} T(l, h_{j+N_i+K}), \tag{11}$$

with $T_d(N_i, h_j) = (N_i + K)T_p$, and $K = 1 + \alpha$ corresponds to the acknowledgment time plus RTT that appears as an addition of one into the time slot indexes and $\alpha \geq 0$ corresponds to the time slots indexes that the RTT occupies. Therefore, $K = 1$ corresponds to the one ACK packet when the RTT is considered to be zero. Additionally, we consider that the probability of erasure of the ACK packet on the reverse link to be zero.

IV. ADAPTIVE NETWORK CODING SCHEME WITH SMART ACKNOWLEDGMENT

Our goal is to find a strategy that allows for a transmission of an optimal number of N packets or degrees of freedom (DoF) that accounts to the channel effects, whether fixed effects or varying over time. Therefore, first, we propose a network coding scheme that accounts for channel variability over time. In particular, we propose a network coding scheme where we transmit N_i coded packets that would account for the channel variations and the DoF at the receiver[2]. If the N_i coded packets are transmitted under certain channel variability, received and decoded successfully, the receiver will send an acknowledgment, asking for the lost degrees of freedom in the first transmission. The new coded packets will be transmitted with new channel states, the process is repeated until all coded packets that are adaptively not transmitted due to channel variability are compensated with encoding across the packets. In particular, the process will be finished when all the DoF are successfully delivered.

An optimal network coded scheme will find the number of coded packets to transmit that can minimize the delay as follows,

$$\begin{aligned} \min_{N_1,\ldots,N_i} T(i, h_j) = \min_{N_1,\ldots,N_i} T_d(N_i, h_j) \\ + \min_{N_1,\ldots,N_i} \sum_{l=1}^{i} P^{N_i}_{(i,h_j)\to(l,h_{j+N_i})} T(l, h_{j+N_i+K}) \end{aligned} \tag{12}$$

Due to the complexity of finding a closed form solution of the optimal number of coded packets to transmit for the scheme in (12), we propose the adaptive scheme, discussed in the following subsection.

A. PHY-Aware adaptive network coded transmission scheme

The strategy of this network coded scheme will rely on the PHY-awareness of the predicted channel. Due to the variation of $P_e(h_j)$ over each packet transmission, the receiver can successfully decode $1 - P_e(h_j)$ packets. Therefore, the adaptive transmission strategy will be to account for the lost packets or lost DoF, via the transmission of coded packets. The following equation presents the proposed adaptive PHY-aware coded transmission scheme:

$$\sum_{s=j}^{N_i^\star} (1 - P_e(h_s)) = i, \tag{13}$$

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[2] Degrees of Freedom (DoF) is the number of linearly independent combinations of the data packets.

where $j = 0$ corresponds to the initial state the timing starts with, which is, $T(N_i, h_0)$. If N_i packets transmitted starting from h_0 to receive successfully i DoF, then for each other transmission we account for the lost DoF by extra coded packets. The system need first to estimate or predict the future channel over a predefined finite interval as discussed before. Then, at each SNR, estimate the probability of erasure vector: $P_e(h_0), P_e(h_1|h_0), ..., P_e(h_L|h_{L-1})$. If the receiver need to receive successfully i DoF, then the transmitter need to account for the packet erasures. This is done by successfully transmitting N_i coded packets; at the point the sum of $(1 - P_e(h_s))$ in (13) is equal to the DoF. The process continues until the last DoF is delivered at the receiver. The adaptive strategy will produce for each SNR a set of optimal number of coded packets to transmit $N^\star(SNR) = [N^\star_{(1)}, ..., N^\star_{(DoF)}]$. The implication of this process, on the mean completion time, is that the transition matrix should span up to $N_i^\star$ in time slots. Therefore, we adaptively transmit accounting for packet erasures caused by channel variation over time and taking into consideration the DoF required at the receiver to be able to decode successfully.

Furthermore, if we analytically compare the proposed adaptive PHY-aware network-coded transmission scheme discussed so far for TVC with the counterpart adaptive PHY-aware network-coded scheme for TIC; then the transmitter will try to adapt the coded packets to a fixed erasure. Therefore, for TIC, such adaptive scheme will produce an optimal number of coded packets that follows,

$$N_i^\star = \frac{N_i}{(1 - P_e)} \tag{14}$$

Where $N_i^\star$ is the optimal number of packets at the transmitter given the fixed probability of erasure P_e, that are required to obtain N_i packets (i DoF) at the receiver side. In principle, we don't care about the order of the packets because we are transmitting linear combinations of coded packets. Therefore, such adaptive schemes are more suited to coded schemes. To clarify the concept, for instance, if a transmission occurs over a channel with 0.2 fixed erasure probability, the transmitter need to account for this by transmitting 1.2 packets over the channel; that is, a 20% more to obtain sufficient linear combinations that allow the receiver to decode successfully.

B. Network Coding Scheme with Smart Acknowledgment

High RTT is known to be a major issue in communications systems, e.g., in satellite communications. A way to partially overcome excessive delays due to high RTT is to allow the system to design smartly the ACK framework. The integration of a smart acknowledgment is associated with the pre-knowledge and estimation of the optimal number of coded packets. In the proposed schemes so far, the delay is introduced due to acknowledging each batch of transmitted packets. However, a smart design will consider an on/off acknowledgment scheme to certain batches and not others. In fact, if we design our system to acknowledge only specific packets like odd or even ones, instead of acknowledging each packet, we can choose the strategy that encounters less delays based on our transmissions if odd or even.

To clarify, if the adaptation strategy provides an optimal number of coded packets to transmit at a certain SNR; $N^\star(SNR) = [N_1^\star, N_2^\star, N_3^\star]$, then we can acknowledge even coded packets $N_2^\star$ and by this we save 2RTT, or we acknowledge odd coded packets $N_1^\star$ and $N_3^\star$ saving one RTT. However, if no adaptation strategy is preceding the smart ACK scheme, the scheme become adaptive to the fixed realization, and so a fixed acknowledgment strategy can be adopted to acknowledge all batches, to do even ACK, or odd ACK.

V. SIMULATION RESULTS

We shall now present a set of results to provide further insight into the performance of the proposed schemes. We consider a packet length $T_p = 1/150$ sec and the log-normally generated channel with mean $m = -0.5$ and variance $\sigma = 1$.

A. Comparison between network coding for TIC and TVC

First, we analyze the delay and throughput with respect to the SNR under different correlation coefficients to compare the network coding scheme for TVC under slow and fast variation to the TIC with fixed erasure.

Figure 3 shows the throughput vs. the SNR, and Figure 4 shows the delay vs. the SNR. Figure 3 clearly depicts the maximum throughput reached in each case which is equal to $1/T_p$. We see that in the case when $a = 0$ and the channel variation is fast, and so the delay is highest, throughput is lowest. However, when $a = 1$ the channel variation is slow, so the delay is lowest, and throughput is highest. Intuitively, we can understand that the main factor influencing the process is the correlation coefficient. In fact, the change in the knowledge in the first channel state has less influence.

Comparing the network coding scheme for TVCs to network coding with fixed erasure, we can see that the assumption of fixed erasure probability is relevant as it gives an average or near to very slow variation behavior of the channel, i.e., when $a = 1$. However, it doesn't give an accurate view of the channel variation and the losses in the throughput that are associated with the delay rise due to fading, particularly at low SNRs.

B. Effect of Round Trip Delay and Smart Acknowledgment

Figure 5 and Figure 6 illustrate network coded scheme over TVC under different RTT. To illustrate the benefit of smart acknowledgment alone, we present here its effect with the non-adaptive scheme with a fixed number of

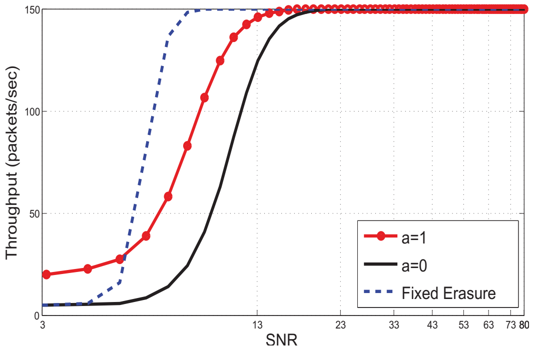

Figure 3. Throughput vs. SNR and channel correlation a.

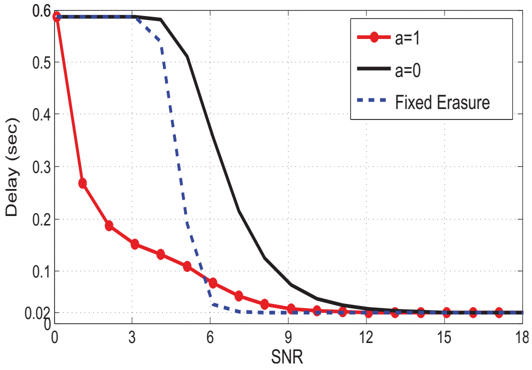

Figure 4. Delay vs. SNR and channel correlation a.

coded of packets to transmit over all SNR levels given by $[N_1, N_2, N_3] = [1, 2, 3]$ and we test three different RTT: 0, 2Tp, 5Tp. Its straightforward to notice that the time of the ACK have significant impact on the delay-throughput performance. For instance, for an odd number of packet transmission, we can see that acknowledging even packets encounters less delays and higher throughputs. Its clear that for such example, around 3 fold of throughput and delay gains can be obtained via adaptive smart acknowledgment. A mixture of both adaptations allows for much more gains, in particular at the low SNRs.

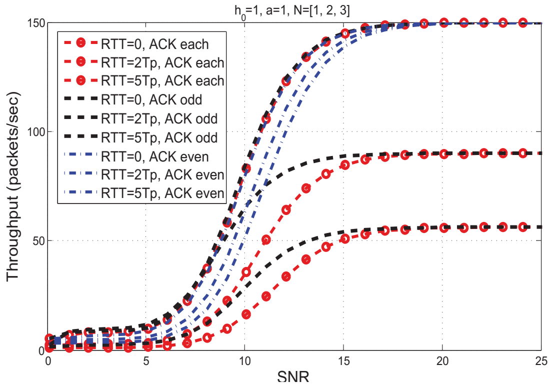

Figure 5. Throughput vs. SNR under different RTT, with odd and even ACK.

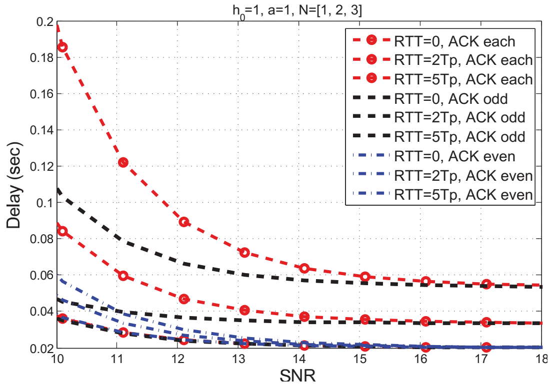

Figure 6. Delay vs. SNR under different RTT, with odd and even ACK.

Its also worth to observe that there is a clear tradeoff between the gains in throughput and reliability that defines which packets to acknowledge. Therefore, if the data is delay sensitive, a broadcast with minimal number of ACKs given the optimal number of coded packets can be selected relying on the reliability introduced via the coding layer. However, if the data is not that delay sensitive, a conservative approach can be used in the acknowledgments of the coded batches.

VI. CONCLUSION

We proposed models and PHY-aware adaptive network-coded transmission schemes with smart acknowledgment for TIC and TVC. We established the fact that with a transmitter-receiver pair that own predictive capabilities, transmission schemes can be adaptive to cross layer awareness and therefore maintain better utilization of the time resources, reducing delay and maximizing throughput in environments with large RTT and complex fading behavior.

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